

## ANALYTICAL STUDY OF $\text{SiO}_2$ -WATER BASED NANO FLUID FLOW IN A ROTATING FRAME WITH THERMO PHORESIS

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### ABSTRACT

*The problem of unsteady MHD free convection flow of  $\text{SiO}_2$ -  $\text{H}_2\text{O}$  nano-fluid through a porous medium bounded by a vertical semi-infinite permeable flat plate with constant heat source and convective boundary condition in a rotating frame of reference is performed, theoretically. The plate is assumed to oscillate in time with steady frequency, so that the solutions of the boundary layer are the same oscillatory types. The entire system rotates about the axes normal to the plate. The dimensionless governing equations for this investigation are solved analytically using perturbation approximation. The effects of various parameters on the flow, heat and mass transfer characteristics are discussed through graphs.*

**KEYWORDS:** Convective Flow, Porous, Rotating Frame, Nano Fluid & Mass Transfer

Original Article

**Received:** May 07, 2019; **Accepted:** May 27, 2019; **Published:** Jul 03, 2019; **Paper Id.:** IJMPERDAUG201943

### INTRODUCTION

The exponential growth of the study of nano fluids made the latest technology more convenient and user friendly. As the Nano fluids carries the metallic particle, the heat transfer and diffusion together is an interesting phenomenon with wide application in the field of biomedical, drug delivery etc. There are many applications of hydromagnetic flow of non-Newtonian fluids in a rotating body in metrology, geographic, turbomachinery, astrophysical and several other areas. The study of Nano fluid attracted many researches since last few decades ([12], [13], and [14]) because of its vital applications. In addition, it has many applications in biomedical such as blood flow in capillaries, dialysis of blood in artificial kidney and flow in blood oxygenation. Rotating flows of MHD fluids have many applications in meteorology, geophysics, turbo machinery and many other fields. Such flows in the presence of a magnetic field are significant because of their geophysical and astrophysical importance and invested by many researchers ([7], [8]). The transport phenomena in porous media have been of continuing interest for the past decades. This is due to its wide applications in solar receiver devices, building thermal insulation, heat exchangers, energy storage units, ceramic processing, and catalytic reactors to name a

few. Furthermore, nanofluids have been also used in porous media due to their superior thermal characteristics. Porous media increase the contact surface area between liquid and solid surface, and, on the other hand, nano particles dispersed in nanofluid enhance the effective thermal conductivity. On the other hand, utilizing porous media in heat exchangers is another technique to augment of thermal efficiency. Porous media by providing high surface area contact will ameliorate heat transfer rate in ducts. The natural convection of boundary layer of a porous medium Nano fluid problems were made by ([3], [9]). Over last decades, numerous works have reported ([1], [6], [11]) on the thermal conductivity of nano fluids than that of the conventional heat transfer fluids with the reason of random motion of nanoparticles. The heat source/sink effects in thermal convection are significant, where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Khan and Aziz [5] represented the boundary layer flow of a Nano fluid past, a vertical surface with a constant heat flux. Generally, it was known that heat and mass fluxes were created from temperature and concentration gradient, respectively. However, heat flux is actually can existed due to the concentration gradient which is known as Soret effect. Same goes to the mass flux, where the flux occurred by the temperature gradient and is called Dufour effect. Configuration involving the heat and mass transfer with Soret and Dufour effects is an important subject due to a wide range of applications such as the solidification of binary alloys, groundwater pollutant migration, chemical reactors, geosciences multi-component melts, oil-reservoirs, isotope separation, and in mixture between gases. Generally, the effects of diffusion of matter caused by temperature gradients (Soret effect) and diffusion of heat caused by concentration gradients (Dufour effect) can be become influential when the temperature and concentration gradients are very large and discussed by many researchers recently ([2], [4], [10]).

Present paper deals the study to analyze the development of unsteady, free convection flow of a Nano fluid, past a moving vertical permeable semi-infinite flat plate in a rotating frame of reference. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency, in the presence of a transverse magnetic field.

## MATHEMATICAL MODEL AND SOLUTION OF THE PROBLEM

We consider an unsteady and three-dimensional flow of an electrically conducting incompressible Nano fluid past a semi-infinite vertical permeable plate. The flow is assumed to be in the  $x^*$  – direction, which is taken along the plate in the upward direction and  $z^*$  axis is normal to the plate.

- A uniform external magnetic field  $B_0$  is taken to be acting along the  $z^*$  axis.
- It is assumed that there is no applied voltage, which implies the absence of an electric field.
- It is assumed the whole system is rotated with a constant vector  $\Omega$  about the  $z^*$ -axis.
- The fluid is grey, absorbing emitting but not scattering medium.
- The radiation heat flux in  $x^*$ -direction is considered negligible in comparison that in the  $z^*$ - direction.
- Due to semi-infinite plate surface assumption, the flow variables are functions of  $z$  and time  $t$  only.
- Assumed that the regular fluid and the suspended nano particles are in thermal equilibrium, and no slip occurs between them.

Boundary layer approximations, the boundary layer equations governing the flow, temperature and concentration along with the Boussinesq are:

$$\frac{\partial w^*}{\partial z^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2\Omega v^* = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u^*}{\partial z^{*2}} + \frac{[\rho\beta]_{nf}}{\rho_{nf}} g(T - T_\infty) - \frac{1}{\rho_{nf}} \sigma B_0^2 u^* - \frac{\mu_{nf}}{\rho_{nf}} \frac{u^*}{k} \quad (2)$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega u^* = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{1}{\rho_{nf}} \sigma B_0^2 v^* - \frac{\mu_{nf}}{\rho_{nf}} \frac{v^*}{k} \quad (3)$$

$$\frac{\partial T}{\partial t^*} + w^* \frac{\partial T}{\partial z^*} = \alpha_{nf} \frac{\partial^2 T}{\partial z^{*2}} - \frac{Q^*(T - T_\infty)}{[\rho C p]_{nf}} \quad (4)$$

$$\frac{\partial C}{\partial t^*} + w^* \frac{\partial C}{\partial z^*} = D_B \frac{\partial^2 C}{\partial z^{*2}} - K_I(C - C_\infty) + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial z^{*2}} \quad (5)$$

The boundary conditions for the problem are given by

$$u^*(z^*, t^*) = 0, v^*(z^*, t^*) = 0, T = T_\infty, C = C_\infty \text{ for } t^* \leq 0 \text{ and any } z^* \quad (6)$$

$$\left. \begin{aligned} u^*(\infty, t^*) &= U_0 \left[ 1 + \frac{\varepsilon}{2} (e^{\ln^* t^*} + e^{-\ln^* t^*}) \right], \quad v^*(\infty, t^*) = 0, -k_{nf} \frac{\partial T}{\partial z^*} = h_f (T_w - T_\infty), C = C_w \\ \text{at } z^* &= 0 \\ u^* \rightarrow 0, v^* \rightarrow 0, T &\rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z^* \rightarrow \infty \end{aligned} \right\} \text{ for } t^* \geq 0 \quad (7)$$

Here,  $u^*$ ,  $v^*$  and  $w^*$  are the velocity components along the  $x^*$ ,  $y^*$  and  $z^*$  axis, respectively.

The properties of nanofluids which are given by (Oztop and Abu-Nada [15])

$$\begin{aligned} \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s, (\rho C p)_{nf} = (1 - \phi)(\rho C p)_f + \phi(\rho C p)_s, \\ (\rho\beta)_{nf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s, \mu_{nf} = \frac{\mu_f}{[1 - \phi]^{2.5}}, \alpha_{nf} = \frac{K_{nf}}{(\rho C p)_{nf}}, \\ \frac{K_{nf}}{K_f} &= \left[ \frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)} \right] \end{aligned} \quad (8)$$

The thermo physical properties of the base fluid (water) and other nanofluids are shown in Table 1.

**Table 1: Nano Fluids and their Thermo Physical Characteristics**  
(Mohammad Mehdi Keshtekar Et Al. [16])

Nanofluid	Density( $\rho$ )	Specific Heat( $C_p$ )	Thermal Conductivity( $k$ )	$\beta \times 10^{-5}$
SiO <sub>2</sub>	3970	765	0.63	36.0
Pure water	997.1	4179	0.613	21

The solution of Equation (1) is considered as

$$w^* = -w_0 \quad (9)$$

Introducing dimensionless variables in the following manner:

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{U_0}, z = \frac{z^* U_0}{v_f}, t = \frac{t^* U_0^2}{v_f}, n = \frac{v_f n^*}{U_0^2}, S = \frac{w_0}{U_0}, R = \frac{2\Omega v_f}{U_0^2}, Q_H = \frac{Q^* v_f^2}{K_f U_0^2},$$

$$Pr = \frac{v_f}{\alpha_f}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \psi = \frac{C - C_\infty}{C_w - C_\infty}, M = \frac{\sigma B_0^2 v_f}{\rho_f U_0^2}, K = \frac{k \rho_f U_0^2}{v_f^2}, Kr = \frac{K_l v_f}{U_0^2}, Sc = \frac{v_f}{D_B},$$

$$Sr = \frac{D_m k_T (T_w - T_\infty)}{T_m v_f (C_w - C_\infty)}, U_0^3 = g \beta_f (T_w - T_\infty) v_f$$

After reducing dimensionless form and the solutions are given by

$$H(z, t) = (1 - B_2) e^{-d_7 z} + B_2 e^{-d_1 z} + \frac{\mathcal{E}}{2} \left\{ e^{-d_8 z} e^{\text{int}} + e^{-d_9 z} e^{-\text{int}} \right\} \quad (11)$$

$$\theta(z, t) = C_1 e^{-d_1 z} \quad (12)$$

$$\psi(z, t) = (1 + B_1) e^{-d_4 z} - B_1 e^{-d_1 z} + \frac{\mathcal{E}}{2} \left\{ d_8 e^{\text{int}} + d_9 e^{-\text{int}} \right\} \quad (13)$$

Where  $H(z, t) = u(z, t) + iv(z, t)$

## RESULT ANALYSIS

A theoretical study on the effect of nano particle SiO<sub>2</sub> on MHD free convection flow along a vertical semi-infinite flat plate with constant heat source, solet effect and chemical reaction when the plate oscillates in time  $t$  in the presence of a rotating frame of reference has been performed. The effects of nanoparticles on the velocity, the temperature and the concentration profiles as well as on the skin friction coefficient, the local Nusselt number and the local Sherwood number are discussed. Figure 1 (a-c) shows the effect of suction parameter  $S$  on the fluid velocity, temperature and concentration for SiO<sub>2</sub> nano particles with  $\phi = 0.15$ . As an output of figures, it is understandable that the velocity, temperature and concentration of the fluid across the momentum boundary layer decreases by increasing suction parameter  $S$  ( $>0$ ). This indicates the usual fact that the suction stabilizes the boundary growth. These consequences are obviously supported from the physical point of view. Figure 2 (a-c) shows the nano fluid velocity, temperature and concentration profiles for different values of heat generation parameter  $Q_H$  with SiO<sub>2</sub> nanoparticles. From the graphs, it is clear that there is a decrease in the nanofluid velocity and

temperature profiles and reverse flow happened in case of concentration with an increasing in  $Q_H$ . These profiles satisfy the far field boundary conditions, asymptotically. This is due to the fact that when heat is absorbed, the buoyancy forces decrease which retard the flow rate and thereby give rise to a decrease in the velocity and temperature profiles. Figure 3 (a-c) illustrates the effect of the nanoparticle volume fraction  $\phi$  on the velocity, temperature and concentration profiles for  $\text{SiO}_2$  nanoparticles. It is clear that as the nanoparticle volume fraction increases, the nanofluid velocity and concentration decreases and the fluid temperature increases with an increasing value of volume fraction parameter  $\phi$ . Hence, the thermal boundary layer thickness increases and tends asymptotically to zero as the distance increase from the thermal boundary layer. This agrees with the physical behavior that, when the volume fraction of  $\text{SiO}_2$  increases, the thermal conductivity increases, and then the thermal boundary-layer thickness increase. Figure. 4 (a-c) demonstrates the effect of the convective parameter  $\gamma$  on the fluid velocity, temperature and concentration. As an output of figures, it is observed that the velocity is increasing, and in the case of the temperature and the concentration, it happened in reverse. From the temperature distribution we have, the fluid within the thermal boundary layer increases with increasing values of  $\gamma$ . Figure 5 (a-c) shows the variation of Prandtl number of nanofluids on velocity, temperature and concentration with  $\phi = 0.15$ . The Prandtl number of nanofluids is found to decrease in velocity and temperature and increase in concentration with an increasing of Prandtl number. The influence of rotation  $R$  parameter on the nanofluid velocity profiles for various values of  $\text{SiO}_2$  nanoparticles with  $\phi = 0.15$ (nanofluid) is shown in figure.6. From the figure, we see that an increasing in  $R$  leads to decreasing in the values of the velocity across the momentum boundary layer. Figure.7 demonstrates the effect of magnetic field parameter  $M$  on the fluid velocity  $\text{SiO}_2$  nanoparticles. It is clear from figure that the velocity distribution across the momentum boundary layer reduces with an increase in the magnetic field parameter  $M$  and decreases asymptotically, to zero at the edge of the hydrodynamic boundary layer. Hence, the hydrodynamic boundary layer thickness decreases as the magnetic field parameter  $M$  increases, the local velocity also decreases. The variation in the concentration boundary layer of the flow field is shown in figure.8. Due to the change in the Soret number,  $Sr$  curves are with  $Sr = 1, 2, 3$  and  $4$ , respectively. Comparing the curves of the said figure, it is observed that a growing Soret number increases the concentration profiles as well as concentration boundary layer thickness.

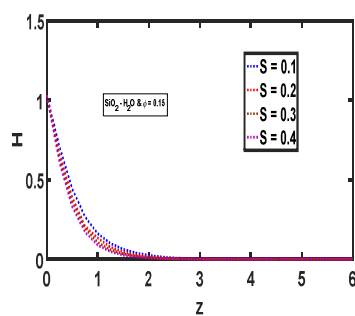


Figure 1(a): Suction Parameter  $S$  Verses  $H$

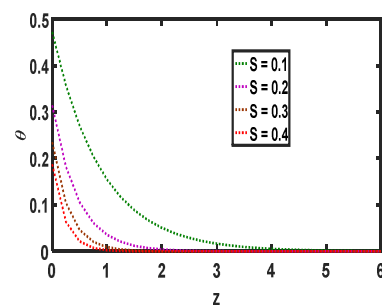
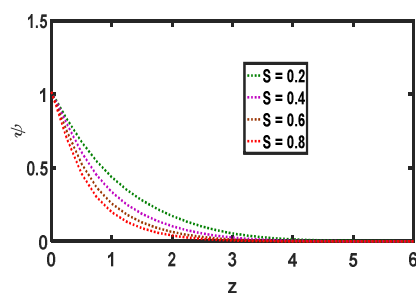
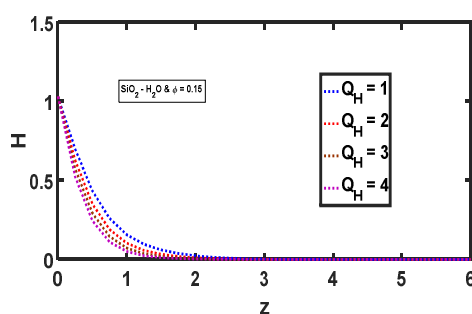
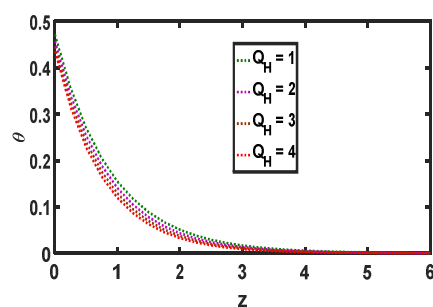
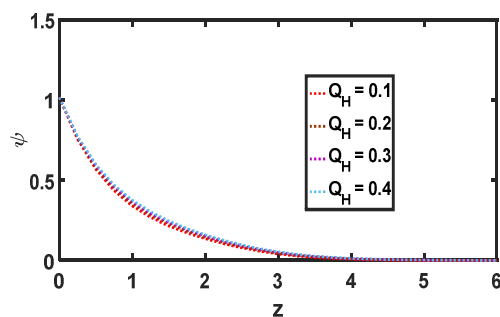
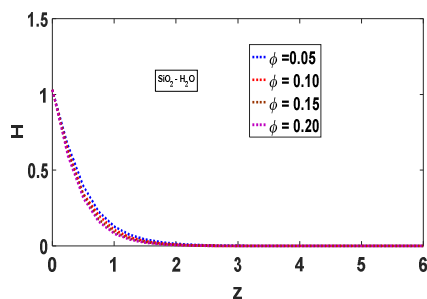
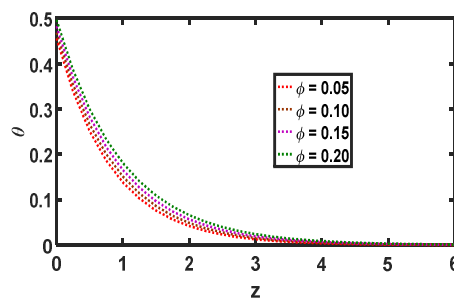


Figure 1(b): Suction Parameter  $S$  Verses  $\Theta$

Figure 1(c): Suction Parameter  $S$  Verses  $\Psi$ Figure 2(a): Heat Generation Parameter  $Q_H$  Verses  $H$ Figure 2(b): Heat Generation Parameter  $Q_H$  Verses  $\Theta$ Figure 2(c): Heat Generation Parameter  $Q_H$  Verses  $\Psi$ Figure 3(a): Volume Fraction Parameter  $\Phi$  Verses  $H$ Figure 3(b): Volume Fraction Parameter  $\Phi$  Verses  $\Theta$

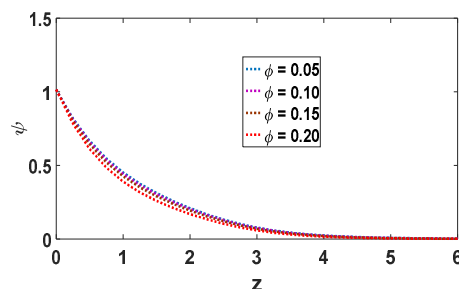


Figure 3(c): Volume Fraction Parameter  $\Phi$  Verses  $\Psi$

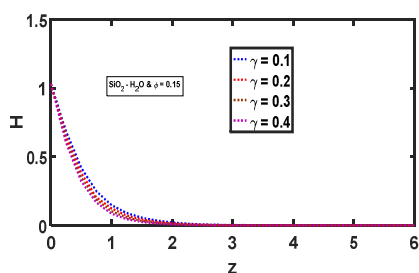


Figure 4(a): Convective Parameter  $\Gamma$  Verses  $H$

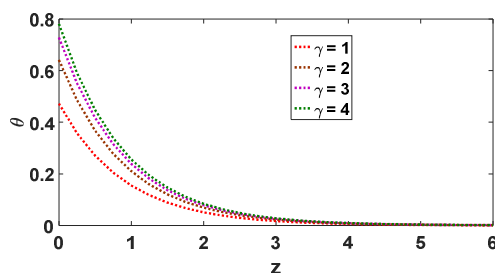


Figure 4(b): Convective Parameter  $\Gamma$  Verses  $\Theta$

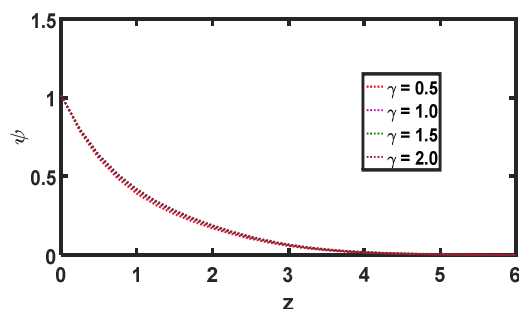


Figure 4(c): Convective Parameter  $\Gamma$  Verses  $\Psi$

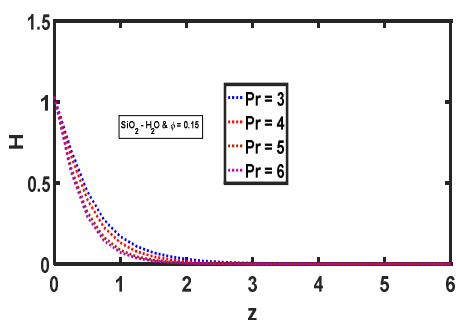


Figure 5(a): Prandtl Number  $Pr$  Verses  $H$

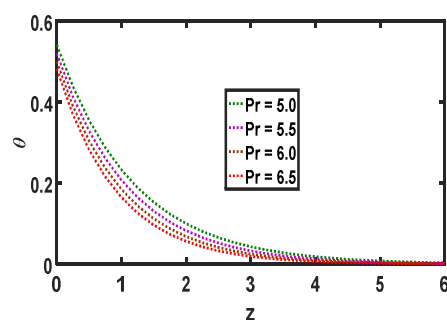


Figure 5(b): Prandtl Number  $Pr$  Verses  $\Theta$

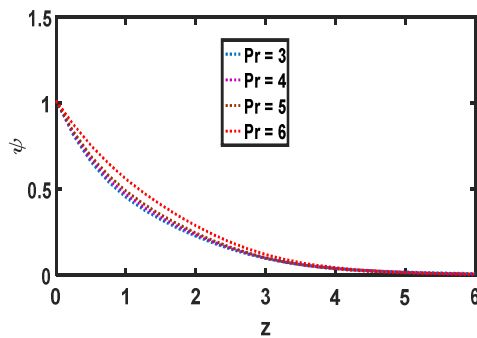
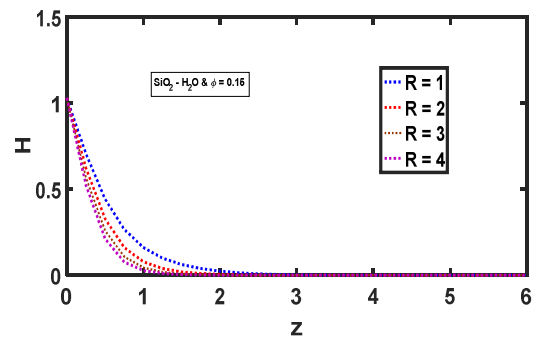
Figure 5(c): Prandtl Number Pr Verses  $\Psi$ 

Figure 6: Rotation Parameter R Verses H

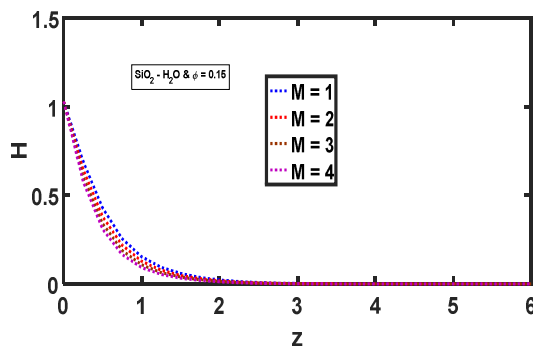


Figure 7: Magnetic Parameter M Verses H

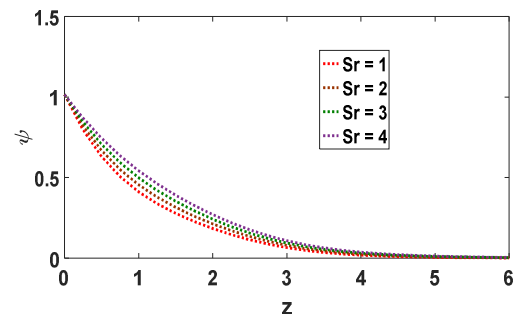


Figure 8: Soret Number Sr Verses

## CONCLUSIONS

In this work, we have studied and analyzed the development of the unsteady free convection flow of a  $\text{SiO}_2$ -water nanofluid past a moving vertical permeable semi-infinite flat plate, in a rotating frame of reference. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency, in the presence of a transverse magnetic field. The effects of various parameters on velocity, temperature and concentration profiles are discussed through graphs. The following are the conclusions derived from the present study.

- As S increases, the  $\text{SiO}_2$ -water nanofluid velocity decreases.
- As the M increases, the  $\text{SiO}_2$ -water nanofluid velocity decreases.
- The nanofluid velocity and concentration decreases, and the fluid temperature increases with an increasing value of volume fraction parameter  $\phi$ .
- There is a decrease in the  $\text{SiO}_2$ -water nanofluid velocity and temperature profiles, and reverse flow happened in case of concentration with an increasing in  $Q_H$ .



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